MODELING GENERALIZED COAXIAL PROBES IN RECTANGULAR WAVEGUIDES

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ABSTRACT

An new approach combining the orthogonal expansion method for modeling cylindrical posts in rectangular waveguides and the extension of the three short plane measurement technique is used to obtain the S-matrix for a general coaxial probe in a rectangular waveguide. The computed results are in excellent agreement with the measured results showing the usefulness of this method.

I. INTRODUCTION

Coaxial probes in rectangular waveguides have been widely applied in a variety of microwave devices as coupling structures or adaptors. Two commonly encounted configurations are coaxial line-waveguide T-junctions and coaxial line-waveguide transitions (coaxial probe excited semi-infinite waveguides). The two configurations are closely related.

Over the years, considerable effort has been made to solve the problems [1-4]. In analysis in [1], a variational method was used to obtain the expression of input impedance for a coaxial line and waveguide transition by assuming a filamentary current located at the center of the probe. This assumption leads to inaccurate results for a thick probe. Using multifilament current approximation in the method of moment, [3] improved the accuracy. In [2], an image theory was employed to develop a closed form expression of input admittance for a hollow probe in rectangular waveguide. A dielectric coated probe in waveguide was studied in [4]. Similar to [1], both [3] and [4] are based on the assumption that the current on the surface of the probe is uniform in angular distribution.

All the above analyses were focused on computing the input impedance (admittance) of a coaxial probe in a rectangular waveguide. However, in many applications, for instance, in designs of probe excited cavity filters and coax-waveguide T-junction manifold multiplexers, it is essential to have the S-parameters of the transitions and the T-junctions. In particular, in the case of a probe located in a section of evanescent mode waveguide, such as a probe excited evanescent mode cavity filter, it is necessary to know the generalized S-parameters of the junction between the coaxial probe and the waveguide in order to design the probe accurately. Limited investigation has been published for such cases. In [5], two-port S-parameters of a coaxial line to waveguide transition was analyzed by three cavity moment method assuming one propagating mode in both the waveguide and the coaxial line. This analysis is valid for a thin probe with simple shape and is incapable of computing generalized S-parameters.

In this paper, an new approach combining the orthogonal expansion method [6,7] and the extension of the three short plane measurement technique [8-10] is applied to model the generalized S-parameters for the more general structures of coax-waveguide T-junctions and transitions shown in Fig. 1. The generalized S-parameters are obtained for all the higher-order modes present in rectangular waveg-

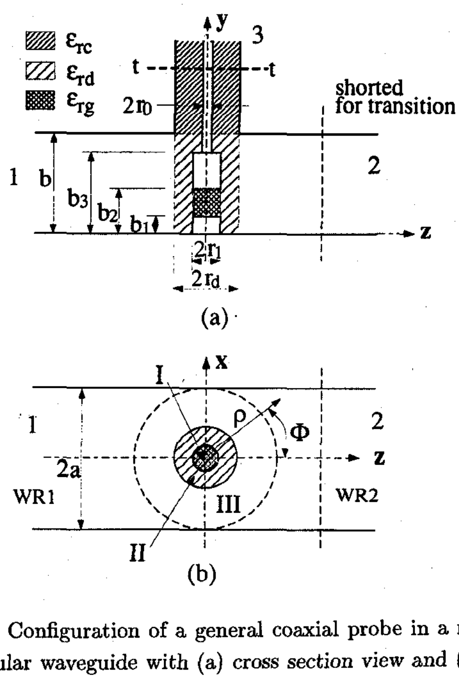

Fig. 1 Configuration of a general coaxial probe in a rectangular waveguide with (a) cross section view and (b) top view

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uides, but only for dominant mode in coaxial lines. The numerical results are compared with the measured results for both T-junctions and transitions. Excellent agreement is achieved.

II. MODELING PROCEDURE

To illustrate the modeling method, we use a T-junction as an example. Modeling of a transition can be obtained from modeling of a T-junction by terminating one of the waveguide ports by a short plane. T-junction's modeling can be separated into two parts: (1) modeling of the generalized S-parameters of the two-port network when the coaxial port of the T-junction is shorted at certain place, and (2) extracting the three-port generalized S-parameters from the obtained two-port S-matrices.

A. Modeling of General Cylindrical Post in Rectangular Waveguide

When the coaxial port is shorted at the position t-t (Fig. 1), following the procedure used in [6] and [7], the resulting two-port network can be divided into waveguide region "WR" and post interaction region "PR" by introducing an artificial cylindrical boundary [12] at $\rho = a$. The post interaction region can be further divided into subregions according to the natural cylindrical boundaries of the geometry. In the case studied, the subregions are: region I ($0 \leq \rho < r_1$), region II ($r_1 \leq \rho < r_d$), and region III ($r_d \leq \rho < a$). Each of these regions can be treated as a multilayer parallel plane bounded in y-direction in which the eigenfunctions can be solved analytically. Then, similar to [11], the boundary conditions on all the natural cylindrical boundaries are forced to be satisfied. By taking proper inner products, one may finally obtain a matrix equation in the form

$$\left[\left[M_C^{III}\right]\left[M_D^{III}\right]\right]\begin{bmatrix}\mathbf{C^{III}}\\ \mathbf{D^{III}}\end{bmatrix} = 0 \qquad (1)$$

where $\mathbf{C^{III}}$ and $\mathbf{D^{III}}$, vectors of size N^{III} (N^{III} represents the number of eigenmodes used in region III), are the field coefficients of the eigenmodes in region III related to the inner-going and the outer-going cylindrical waves, respectively. $\left[M_C^{III}\right]$ and $\left[M_D^{III}\right]$, matrices of size $N^{III} \times N^{III}$, can be grouped as diagonal block matrices in terms of ϕ variations. This factor can be used to improve the efficiency of matrix operations later on.

The fields in waveguide regions are related to the fields in region III by applying boundary conditions at the artificial boundary. Then, Bessel-Fourier series are used to expand the fields in waveguide regions in terms of ϕ-dependent orthogonal eigenfunctions of the fields in region III. Taking inner products using the fields in region III, the following equations may be obtained

$$\begin{bmatrix}\mathbf{C^{III}}\\ \mathbf{D^{III}}\end{bmatrix} = \begin{bmatrix}[M_A^{11}] & [M_A^{12}]\\ [M_A^{21}] & [M_A^{22}]\end{bmatrix}\begin{bmatrix}\mathbf{A^{(1)}}\\ \mathbf{A^{(2)}}\end{bmatrix} + \begin{bmatrix}[M_B^{11}] & [M_B^{12}]\\ [M_B^{21}] & [M_B^{22}]\end{bmatrix}\begin{bmatrix}\mathbf{B^{(1)}}\\ \mathbf{B^{(2)}}\end{bmatrix} \qquad (2)$$

where $\mathbf{A^{(i)}}$ and $\mathbf{B^{(i)}}$ are vectors of size $N_W^{(i)}$, representing the field coefficients of the incident and reflected waves in waveguide 'i' ($i = 1,\ or\ 2$), respectively. $N_W^{(i)}$ is the number of modes used in waveguide 'i'. The elements of matrix $[M_A]$ and $[M_B]$ are determinated by the inner products.

From equation (1) and (2), the generalized scattering matrix $[S^{t\text{-}t}]$ of the cylindrical post can be acquired as

$$\begin{bmatrix}\mathbf{B^{(1)}}\\ \mathbf{B^{(2)}}\end{bmatrix} = \begin{bmatrix}[S_{11}^{t\text{-}t}] & [S_{12}^{t\text{-}t}]\\ [S_{21}^{t\text{-}t}] & [S_{22}^{t\text{-}t}]\end{bmatrix}\begin{bmatrix}\mathbf{A^{(1)}}\\ \mathbf{A^{(2)}}\end{bmatrix} = [S^{t\text{-}t}]\begin{bmatrix}\mathbf{A^{(1)}}\\ \mathbf{A^{(2)}}\end{bmatrix} \qquad (3)$$

where the superscript 't-t' refers to the short plane t-t.

Equation (3) can be obtained only under the condition of $N^{III} = N_W^{(1)} + N_W^{(2)}$, that is, the number of modes used in region III must be equal to the total number of modes used in both sides of the waveguide. In order to avoid singular matrix when matching the artificial boundary, the eigenmodes used in both region III and the waveguide regions have to be selected carefully. The general rule of selecting the modes is: the numbers of the modes with same y-variations in region III and the waveguide regions must be equal.

B. Extraction of S-matrix of T-Junction or Transition

The S-matrices of a coaxial-waveguide T-junction can be extracted from three two-port S-matrices $[S^{t\text{-}t}]$ resulting from putting a short plane at three different positions t-t ($t = 1,\ 2,\ and\ 3$) in the coaxial port of the T-junction (Fig. 2). Assuming only the dominant TEM mode exists in the coaxial line beyond the position 1-1, then the S-matrix of

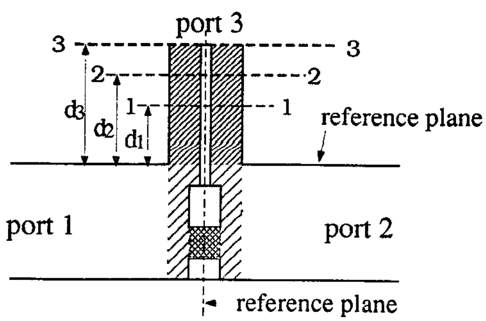

Fig. 2 Coaxial-waveguide T-junction with coaxial port shorted at three different positions

the T-junction has the following form

$$[S] = \begin{bmatrix} [S_{11}]_{N_W^{(1)} \times N_W^{(1)}} & [S_{12}]_{N_W^{(1)} \times N_W^{(2)}} & [S_{13}]_{N_W^{(1)} \times 1} \\ [S_{21}]_{N_W^{(2)} \times N_W^{(1)}} & [S_{22}]_{N_W^{(2)} \times N_W^{(2)}} & [S_{23}]_{N_W^{(2)} \times 1} \\ [S_{31}]_{1 \times N_W^{(1)}} & [S_{32}]_{1 \times N_W^{(2)}} & S_{33} \end{bmatrix} \quad (4)$$

The relationship between $[S]$ and $[S^{t\text{-}t}]$ ($t = 1, 2, and\, 3$) may be derived as

$$S_{33} = \frac{\Delta_{21} \left[S_{11}^{1\text{-}1}\right]_{11} + (\Delta_{12} - \Delta_{32}) \left[S_{11}^{2\text{-}2}\right]_{11} - \Delta_{23} \left[S_{11}^{3\text{-}3}\right]_{11}}{\Gamma_1 \Delta_{21} \left[S_{11}^{1\text{-}1}\right]_{11} + \Gamma_2 (\Delta_{12} - \Delta_{32}) \left[S_{11}^{2\text{-}2}\right]_{11} - \Gamma_3 \Delta_{23} \left[S_{11}^{3\text{-}3}\right]_{11}} \quad (5a)$$

$$[S_{ij}]_{\substack{i \neq 3 \\ j \neq 3}} = \Gamma_1 \Delta_{21} \left(\frac{1}{\Gamma_1} - S_{33}\right) \left[S_{ij}^{1\text{-}1}\right] + \Gamma_2 \Delta_{12} \left(\frac{1}{\Gamma_2} - S_{33}\right) \left[S_{ij}^{2\text{-}2}\right] \quad (5b)$$

$$[S_{i3}][S_{3j}]_{\substack{i \neq 3 \\ j \neq 3}} = \left(\frac{1}{\Gamma_1} - S_{33}\right) \left[S_{ij}^{1\text{-}1}\right] - \left(\frac{1}{\Gamma_1} - S_{33}\right) [S_{ij}] \quad (5c)$$

where $\Delta_{ij} = \frac{\Gamma_i}{\Gamma_i - \Gamma_j}$, and $\Gamma_i = -e^{-j2\beta d_i}$. β is the propagation constant of TEM mode in coaxial line. When $N_W^{(1)} = N_W^{(2)}$, the properties of symmetrical networks can be applied to reduce the computational effort.

III. RESULTS

Computer programs have been developed to calculate the S-matrix of a coaxial-waveguide T-junction or a coaxial waveguide transition. The eigenmodes used in waveguide regions and cylindrical region III are selected according to the rule given in Section II. The numbers of the modes in ϕ-variation in all cylindrical regions are the same, but the number of modes in y-variations in each region is determinated by the ratio of the region height to the height of region III in order to have good convergence. Numerical experiments show that using 5 modes in ϕ-variation and 10 modes in y-variation in region III can provide convergent results in most cases.

Fig. 3 shows the dominant mode scattering parameters of a coaxial-waveguide T-junction in which the dielectric coated coaxial probe is extended from the top to the bottom of the waveguide. The figure also shows the measured results of the S-parameters. The two sets of results are in excellent agreement.

Fig. 4 compares the computed and measured results of the dominant mode S-parameters for a coaxial-waveguide transition with a disc loaded probe. Again, good agreement between the numerical results and the experimental results is achieved.

In Fig. 5, the input impedance $Z_{in} = R_{in} + jX_{in}$ at the coaxial port of a coaxial waveguide transition with a tuning rod from the bottom wall is presented. The input impedance is normalized by the characteristic impedance of the coaxial line (Z_0) and computed from the reflection coefficient at the coaxial port (port 2 in this case).

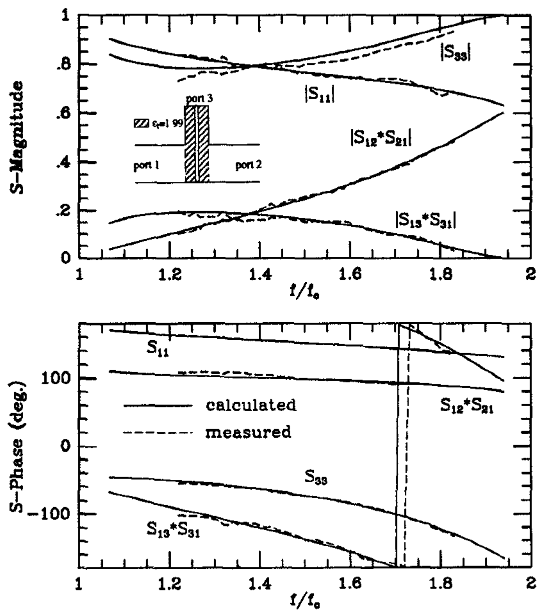

Fig. 3 Dominant mode S-parameters of a coaxial-waveguide T-junction with a dielectric coated probe with b/2a=0.4444, r_0/a=0.0556, and 50 Ω coaxial line

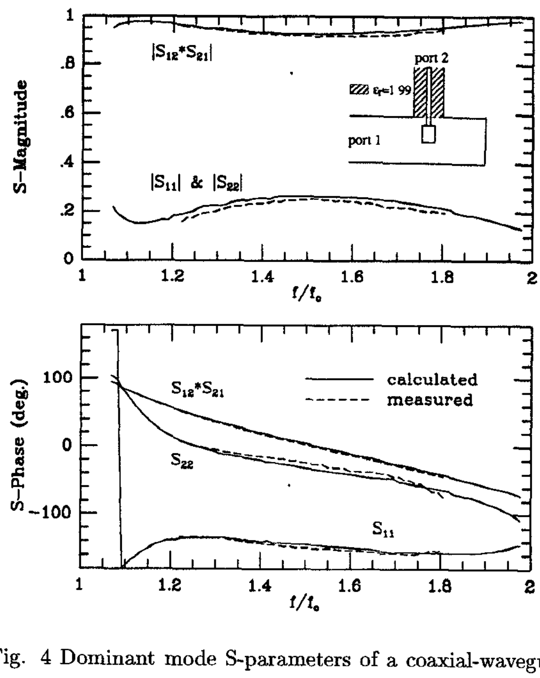

Fig. 4 Dominant mode S-parameters of a coaxial-waveguide transition with a disc loaded probe with b/2a=0.4444, b_1=0, b_2/b=0.425, $(b_3 - b_2)/b$=0.2975, $\ell/2a$=0.3889, ,r_0/a=0.0556, r_d/r_0=2.14, and 50 Ω coaxial line

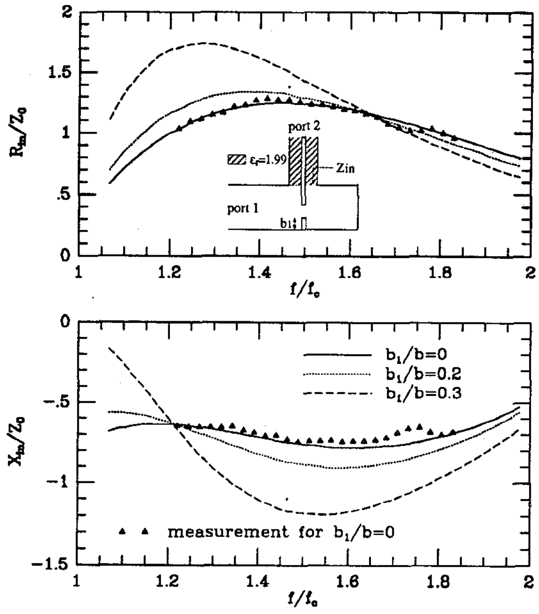

Fig. 5 Input impedance of a coaxial probe excited semi-infinite waveguide with a tuning rod from bottom wall of waveguide with $b/2a$=0.4444, b_2/b=0.425, $\ell/2a$=0.3889, r_0/a=0.0556, and 50 Ω coaxial line

IV. CONCLUSION

An new approach combining the orthogonal expansion method and the extension of the three short plane measurement technique is applied to analyze a general coaxial probe in a rectangular waveguide. Scattering matrices are obtained for both coaxial-waveguide T-junctions and coaxial waveguide transitions. The computed results are in excellent agreement with the measured results for all cases. This method will be very useful in analysis and design of coaxial-waveguide adapters, coaxial probe excited cavity filters, and coaxial-waveguide T-junction manifold multiplexers.

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